Simulation of Slowwave Spiral Structures Based on Analytical Model

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***Abstract*–The authors of the article consider decelerating structures made of homogeneous material, which have a periodic structure in space. Such systems are used to concentrate the energy of high-frequency electromagnetic waves in order to increase the sensitivity of devices designed for their registration, increase the efficiency of the interaction of a beam of free electrons with a slowed electromagnetic field, for the manufacture of structural elements in waveguide devices, and for generators of monochromatic radiation in the terahertz and optical ranges (effect Vavilov-Cherenkov radiation). The parameters of the wave process are studied on the basis of an exact analytical solution based on the cylindrical Bessel and Hankel functions for a decelerating system with axial symmetry in the form of a spiral. To obtain numerical solutions, the optimization problem of the system of nonlinear equations of a complex variable is solved. The conducted studies establish the relationship between the transverse and longitudinal wave numbers and the attenuation coefficient of the electromagnetic wave. A detailed analysis of the solutions of the equation showed that, in addition to the classical solution that determines the surface wave, other solutions are possible for which the concentration of the electromagnetic field inside the structure is higher.**

Keywords–decelerated electromagnetic waves, cylindrical functions, transverse wavenumber, longitudinal wavenumber.

I. Application of Slowwaves Structures

One of the parameters of high-frequency electromagnetic field sensors is the resonance frequency of the interaction between the external electromagnetic field and the sample under study and the phase delay of the electromagnetic wave in a field-sensitive element of the sensor. The structure and length of the sensitive element are the main parameters that determine the sensitivity of the entire device.

Devices with a uniform internal microstructure and characteristic dimensional parameters, which by the order of magnitude coincide with the size of the wave of the investigated field, are classical. Many of them have the property of self-similarity, and therefore can be considered as fractal objects [1-3]. The effect of wave deceleration is quite effectively used in electromagnetic field sensors of the microwave range, where they are used as “feedback elements in oscillator-type circuits in which the slow wave structure acts as a delay line or as a phase-shifting element” [4].

Decelerating structures also find a number of other applications [5]:

- generation of electromagnetic waves by electron beams (Vavilov-Cherenkov effect);
- construction of lasers based on free electron beams with Cherenkov radiation;
- structural elements of elementary particle accelerators;
- generators of ultra-short pulses for radar and lidar equipment;
- miniaturization of waveguide technology components in the microwave range.

Fig. 1a, 1b, 1c and 1d show the geometries of the most common decelerating structures made of a homogeneous conductive material [5].

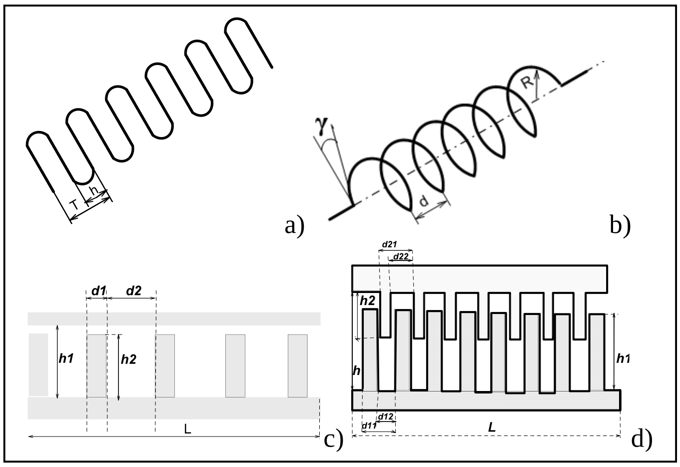

Fig. 1. Decelerating structures made of homogeneous materials: a) meanders; b) spiral; c) combs and d) combs double row.

Devices with a complex microstructure are also the subject of functional materials science research due to the development of nanotechnology [6, 7]. The ability to control the microstructure of devices significantly increases the potential scope of their application, in particular, it enables the transition to the optical range with the simultaneous use of optical sensors [8-10]. This led to the rapid development of the branch of science, which is characterized as the use of materials with close to zero dielectric constant [11, 12].

II. Formulation of the Problem

The previously applied differential-symbolic method allows us to study the propagation processes of electromagnetic waves, which can be modelled by the one-dimensional equation of a telegraph line [13, 14] in conductor structures. However, it is also effective for studying objects and processes that can be modelled by the Helmholtz equation based on equivalent electrical circuits [6].

The most general solution to problems of generation, propagation, absorption and detection of an electromagnetic wave is obtained as a result of solving the system of Maxwell's

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equations with given boundary and/or initial conditions. For all guiding structures having axial symmetry, the system of Helmholtz equations with boundary conditions given on the surface of the structure, which is a conductor, reduces to the two-dimensional Helmholtz equation.

Two types of solutions are considered, which define transverse magnetic (TM) or transverse electric (TE) electromagnetic waves. For the first type of waves with boundary conditions which determing equality to zero value of the electric field on the surface of the conductor or equality to zero derivative of the value of the magnetic field in the direction perpendicular to the surface of the conductor. For the use a structure as a device for slowing down electromagnetic waves, the application where the longitudinal component is the electric component, i.e. the TE wave, prevails.

Thus, the problem of investigating the wave process is reduced to the two-dimensional Helmholtz equation (1) with a zero value of the electric field on the outer surface of the structure:

$$\begin{cases} \Delta T(x,y) + \chi^2 \cdot T(x,y) = 0 \\ \chi^2 = k^2 - \Gamma^2 \\ A(x,y,z) = T(x,y) \cdot Z(z) \end{cases} \tag{1}$$

where $A(x,y,z)$ is the value of the field amplitude, which can be represented by the method of separation of variables as a product of two functions $Z(z)$ and $T(x,y)$, Γ is the longitudinal wave number, which is determined by the periodicity of the structure along the direction of wave propagation, χ is the transverse wavenumber.

Structures with cylindrical symmetry are very important from the point of view of practical application and prevalence. Thus, to determine the characteristics of the field in the decelerating structure with axial symmetry, it is necessary to determine two wave numbers: longitudinal Γ and transverse χ . Wave attenuation is determined by the imaginary parts of the longitudinal and transverse wave numbers. The most important parameter of the slowing wave structure is the transverse wave number: if $\chi^2 < 0$, then there is an effect of slowing down of the electromagnetic wave, and its phase speed becomes less than the wave speed outside the structure (speed of light): $v_{ph} < c$.

The wavelength in the spiral can be estimated according to the model given in [5]. Provided that the distance between adjacent turns of the spiral is very small compared to its radius, we find the wavelength in the spiral:

$$\lambda = \lambda_0 \cdot tg\gamma , \tag{2}$$

where λ_0 is the wavelength in a free space. Thus, the formula for the longitudinal wave number, has the form:

$$\Gamma = \frac{k}{\sin\gamma} , \tag{3}$$

where γ is the angle of the spiral (Fig. 1b).

The more accurate model is based on a solutions of the system (1) in cylinder coordinates. Then the mathematical model of this problem is reduced to an ordinary differential equation of the second order (5), the variable of which is the distance r to the axis of symmetry of the system.

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \cdot \frac{dR}{dr} + \left(\chi^2 - \frac{n^2}{r^2} \right) R = 0 \tag{4}$$

The general solution of this equation is determined by a linear combination of cylindrical Bessel and Newmann functions $J_n(\chi r)$, $N_n(\chi r)$:

$$R(r) = A \cdot J_n(\chi r) + B \cdot N_n(\chi r) \tag{5}$$

where n is the order of these functions, A and B are some constants. За межами структури поле швидко затухає, тому розв’язок задачі має ви

For a spiral structure with axial symmetry (Fig. 1b), the solution of problem (5) is divided into two independent analytical expressions. In order for the expression not to grow indefinitely at $r \to 0$, inside the structure the dependence of the field strength on the radius is determined by the Bessel function:

$$T(r,\alpha) = A \cdot J_n(\chi r) \tag{6}$$

Outside the structure, the solution is defined by Hankel functions, which decay faster as $1/\sqrt{r}$:

$$T(r,\alpha) = C \cdot H_n^{(2)}(\chi r) \tag{7}$$

The condition of non-discontinuity of electric and magnetic field vectors on the surface of the structure makes it possible to obtain a transcendental equation with respect to the transverse wave number χ :

$$\chi^2 = -\frac{J_1(\chi R) \cdot H_1^{(2)}(\chi R)}{J_0(\chi R) \cdot H_0^{(2)}(\chi R)} \cdot k^2 \cdot ctg^2\gamma \tag{8}$$

This equation includes such parameters of the system as radius R and γ, and wave number k .

III. Analysis Of the Wave Process in the Spiral Structure

Expression (8) is a transcendental equation, which includes cylindrical functions of the complex argument: Bessel the first ($J_1(\chi R)$) and of zero order ($J_0(\chi R)$) and Hankel of the second kind of the first ($H_1^{(2)}(\chi R)$) and of zero order ($H_0^{(2)}(\chi R)$).

Further analysis is reduced to the solution of the optimization problem given by the equation, which is a problem for determining the roots of systems of two nonlinear equations (9), which expresses the equality of the phases and modules of the left and right sides of equation (8):

$$\begin{cases} phase(\chi)^2 = phase\left\{-(k\cdot ctg\gamma)^2 \dfrac{J_1[\chi R]\cdot H_1^{(2)}[\chi R]}{J_0[\chi R]\cdot H_0^{(2)}[\chi R]}\right\} \\ abs(\chi)^2 = abs\left\{-(k\cdot ctg\gamma)^2 \dfrac{J_1[\chi R]\cdot H_1^{(2)}[\chi R]}{J_0[\chi R]\cdot H_0^{(2)}[\chi R]}\right\} \end{cases} \quad (9)$$

The new variables of the problem are the module r and the phase φ of the product χR of the transverse number χ and the radius of the spiral R, presented in the exponential form (10):

$$\chi R = r\cdot e^{j\varphi} \quad (11)$$

The properties of the wave process in a wide range of values of the number χR. Analysis of relation (8) shows that the existence of solutions to the problem is determined by the phase of the number χR, provided that the longitudinal wave number k is a real value. The equality of the modules can be achieved by selecting the value of the parameter $(k\cdot ctg\gamma)^2$, which is the square of the product of the longitudinal wavenumber by the cotangent of the spiral rotation angle.

In this way, it is necessary to investigate the difference in the phases of expressions $(\chi)^2$ and $-k^2ctg^2\gamma\cdot\dfrac{J_1[\chi R]\cdot H_1^{(2)}[\chi R]}{J_0[\chi R]\cdot H_0^{(2)}[\chi R]}$. The studies are performed in a wide range of modulus values r (from 10^{-4} to 10^{3}) and phase φ (from $-\pi$ to $+\pi$ with step $\pi/100$).

The ranges of the existence of solutions of the system (10) (with an accuracy of phases equality $\pi/100$) are found and listed in Table 1.

TABLE I. RANGES OF VALUES OF THE MODULUS AND THE PHASE OF THE NUMBER χR FOR WHICH EXISTS SOLUTIONS OF THE PROBLEM (10,11)

$\chi R = r\cdot e^{j\varphi}$	
Modulus r	***Phase*** φ ***, rad***
$10^{-4}\div10^{-3}$	-1.571 (-90 deg); 1.723 (98.7 deg)
$10^{-3}\div10^{-2}$	-1.571 (-90 deg); 1.791 (102.62 deg)
$10^{-2}\div10^{-1}$	-1.571 (-90 deg); 1.885 (108 deg)
0.1	-1.571 (-90 deg); 2.042 (117 deg)
0.2	-1.571 (-90 deg); 2.136 (122.38 deg)
$0.3\div0.4$	-1.571 (-90 deg); 2.168 (124.22 deg)
9	-1.571 (-90 deg); 3.016 (172.80 deg)
$10\div11$	-1.571 (-90 deg); 0.063 (3.61 deg)
$20\div40$	-1.571 (-90 deg)
50	-1.571 (-90 deg); 0.031 (1.78 deg)
$50\div100$	-1.571 (-90 deg)

The obtained phase values are approximate and make it possible to continue the search for a solution to the optimization problem. At this stage of the search for solutions, the entire problem is reduced to determining the values of the second unknown variable of the optimization problem (module r).

For all investigated ranges of the value of the modulus of the transverse wave number, there is a solution when this number has only an imaginary part ($\varphi = -\pi/2 \approx 1.571$). This is a well-known solution for which the problem reduces to the following form:

$$\chi = \frac{k^2ctg^2\gamma}{R}, \quad (12)$$

It is also known [5] that in such a system the greatest density of lines of force is observed near the turns of the spiral, and when moving away from the spiral outwards or towards the axis of symmetry of the structure, it decreases. Thus, the solution corresponding to the imaginary wave number χ determines the surface wave process. Therefore, an important task is the study of other possible variants of the wave process, for which a greater concentration of electromagnetic field energy is observed inside the structure, and at the same time there is an effect of slowing down electromagnetic waves.

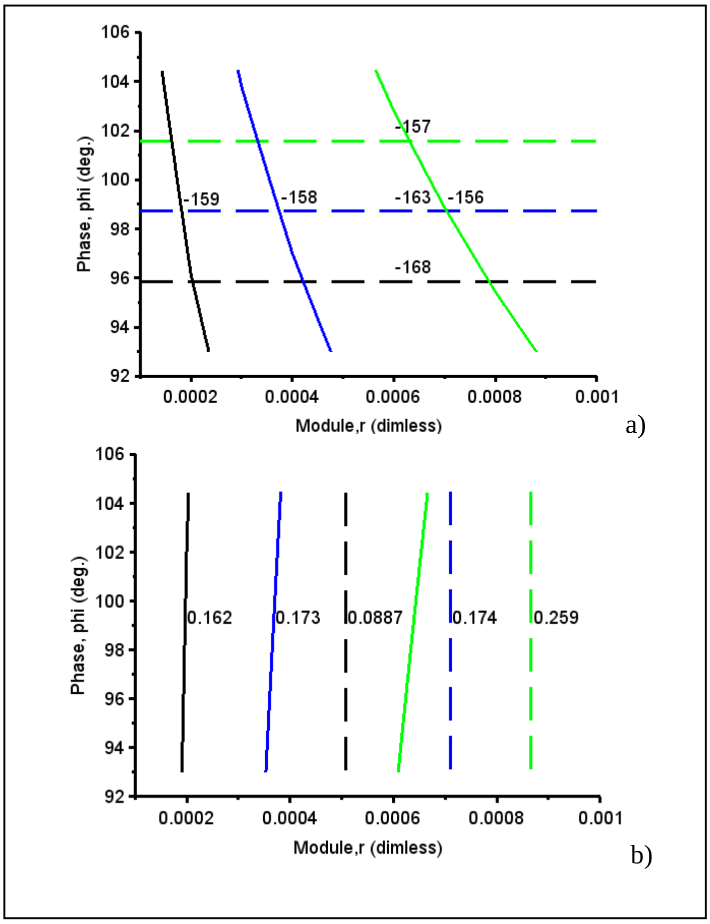

Fig. 2. Contour plots in the plane of the modulus (vertical axis) and phase (horizontal axis) of the χR of the phase left side (dashed line on Fig. 2a)) and right side (solid line on Fig. 2a) of the optimization task (9), and of the module left side (dashed line on Fig. 2b)) and module right side (solid line on Fig. 2b)) of this task

In Fig. 2a and Fig. 2b shows the lines of the equal values of the phase and modulus of the left and right parts of the optimization task (10) in the plane of the module r (horizontal axis) and phase φ (the vertical axis) of the number χR, which are derived for solutions corresponding to the first row of Table 1, namely $\varphi = 1.723$ (99 deg), $r \in 10^{-4} \div 10^{-3}$.

In Fig. 2a, the horizontal dashed lines mean lines of equal phase for $(\chi)^2$, and the solide ones are lines of equal phase for expression $-k^2 ctg^2 \gamma \cdot \frac{J_1[\chi R] \cdot H_1^{(2)}[\chi R]}{J_0[\chi R] \cdot H_0^{(2)}[\chi R]}$.

Fig. 2b shows the lines of equal module for $(\chi)^2$ (dashed line) and for $-k^2 ctg^2 \gamma \cdot \frac{J_1[\chi R] \cdot H_1^{(2)}[\chi R]}{J_0[\chi R] \cdot H_0^{(2)}[\chi R]}$ (solid line).

TABLE II. VALUE OF REAL AND IMAGINE PARTS OF LONGITUDINAL WAVE NUMBER Γ

Transverse wavenumber χ	**Longitudinal wave number** Γ		
	k		
	1	**3**	**5**
$5 \cdot 10^{-3} \cdot e^{1.723j}$	1	3	5
$5 \cdot 10^{-2} \cdot e^{1.791j}$	1	3	5
$5 \cdot 10^{-1} \cdot e^{1.885j}$	$1.1 + 0.07j$	$3.03 + 0.02j$	$5.02 + 0.01j$
$3.5 \cdot e^{2.168j}$	$3.01 + 1.89j$	$3.95 + 1.44j$	$5.53 + 1.03j$
$100 \cdot e^{3.016j}$	$12.53 + 99.21j$	$12.53 + 99.17j$	$12.54 + 99.09j$
$500 \cdot e^{0.031j}$	$15.50 - 500j$	$15.50 - 500j$	$15.50 - 500j$

According to the data in Table 1 and the ratio between the numbers k, longitudinal Γ and transverse χ wave numbers ($\Gamma^2 = k^2 - \chi^2$), the values of the longitudinal wave number are determined for $k = 1 \div 5$.

Table 2 shows the calculated values of the transverse and longitudinal wave numbers for the radius structure $R = 0.1\,м$. As we can see from this table, the values Γ found can have both a real and an imaginary part, while the classical solution for determines only the real transverse wavenumbers.

IV. CONCLUSIONS

The paper analyzes an analytical model that determines the value of transverse wave numbers in a spiral axisymmetric decelerating structure. In contrast to the known classical solution, solutions were found for which the transverse wave number has a real part, and the longitudinal wave number has an imaginary part. This means that in such a structure there can exist not only a surface wave, but also a wave whose energy is concentrated in the volume of the structure, but at the same time the wave is attenuated in the longitudinal direction.

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